Principe du canon magnétique

$$x(t=0)=0 \quad ; \quad v(t=0)=0$$

1) $t=0$ I courant dans le circuit $E = RI$

2) Force de Laplace $\vec{F}_L = \int_{NM} I\,\vec{dl} \wedge \vec{B}$ avec $\vec{dl} = dy\,\vec{e}_y$ $\vec{B} = B\vec{e}_z$

$\Rightarrow$ direction, sens $+\vec{e}_x$

$$|\vec{F}_L| = IlB$$

3) Expression de la vitesse $m\dfrac{dv}{dt} = IlB \Rightarrow v(t) = \dfrac{IlBt}{m}$

4) $x(t)$ tel que $\dfrac{dx}{dt} = \dfrac{IlBt}{m} \Rightarrow x(t) = \dfrac{IlBt^2}{2m}$

5) temps pour quitter les rails $\dfrac{IlBt_R^2}{2m} = h \iff t_R = \sqrt{\dfrac{2mh}{IlB}}$

6) $\mathcal{E}$ gagnée : $\Delta\mathcal{E}_R = \frac{1}{2}mv_R^2 = \frac{1}{2}m\frac{I^2l^2B^2}{m^2}\times\frac{2mh}{IlB} = IlBh$

Résonateur de Helmholtz

1) RFD $\underbrace{m}_{\rho V = \rho s l} \dfrac{d^2x(t)}{dt^2} = \underbrace{F}_{s(p-p_0)} \iff \rho s l \dfrac{d^2x(t)}{dt^2} = s(p-p_0)$

$\alpha = -1$

2) Laplace $pV^\gamma = cste \Rightarrow$ différenciation :

$\ln(pV)^\gamma = cste' = \ln p + \gamma \ln V \Rightarrow \dfrac{dp}{p} + \gamma \dfrac{dV}{V} = 0$

$\Rightarrow \dfrac{(p-p_0)}{p_0} + \gamma \dfrac{s\,x(t)}{V_0} = 0$

$\beta = \gamma$

3) $\rho s l \dfrac{d^2x(t)}{dt^2} = -s p_0 \gamma s \dfrac{x(t)}{V_0}$

$\iff \rho s l \dfrac{d^2x(t)}{dt^2} + \gamma \dfrac{s^2 p_0}{V_0} x(t) = 0$

$\iff \dfrac{d^2x(t)}{dt^2} + \underbrace{\gamma \dfrac{s p_0}{\rho l V_0}}_{\omega_0^2} x(t) = 0 \Rightarrow \omega_0^2 = \gamma \dfrac{s p_0}{\rho l V_0}$

4) $f_0 = \dfrac{\omega_0}{2\pi} = \dfrac{1}{2\pi}\sqrt{\gamma \dfrac{s p_0}{\rho l V_0}}$

AN : $\gamma = 1{,}4$

$s = 5\,(10^{-2}\,m)^2 = 5\,10^{-4}\,m^2$

$p_0 = 101325\ Pa$

$\rho = 1{,}3\ kg\,m^{-3}$

$l = 5\,10^{-2}\,m$

$V_0 = 10^{-3}\,m^{+3}$

AN : $f_0 = 166{,}25\ Hz$

5) $\lambda = \dfrac{c_0}{f_0} \Rightarrow$ AN : $\lambda = 2{,}05\ m$ à comparer à $R = \sqrt[3]{\dfrac{3V_0}{4\pi}}$ AN : $R = 6{,}2\ cm$

$\Rightarrow$ grand